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SON OF SUPERNOVA 1987a: THUNDER AND LIGHTNING?

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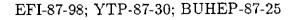
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Abstract

Using the recent speckle interferometry data on the companion of SN 1987a, we construct a set of physical criteria that models for the companion must satisfy. We consider mechanisms including: (a) a reflection nebula, (b) dense matter ejection, (c) jet interactions with external media, and propose a fourth mechanism based on magneto-hydrodynamic effects (analogous to SS4330). If the companion is observed to move at high velocity with roughly constant luminosity, then it will be difficult to reconcile observations with any but the latter model.

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Supernova 1987a may be continuing to provide new cosmic fireworks some 8 months after its initial observation. Four months ago, using speckle imaging techniques, two groups reported the observation of a companion to SN 1987a with a luminosity far exceeding any star in the pre-supernova visual field.[1,2] This progeny, which we will refer to as the Son of Supernova (SOS), cries out for an explanation. In this paper we examine a wide variety of models, ranging from projectiles to jets, and use constraints derived from the obvservational evidence to establish their consistency. The present data is inconclusive in distinguishing models. In particular, the *non-observation* of the SOS in the background of SN 1987a in June is consistent either with a constant luminosity (since SN 1987a itself had increased in luminosity since the first observation) or a rapidly falling luminosity. If, however the companion is in fact moving at high velocity with constant luminosity [3], then we suggest a model, based on electromagnetic effects. which may be more easily reconciled with this possibility than any of the other models we discuss. Future measurements will be necessary before this possibility, which motivated our initial investigations, can be definitively tested.

1. The Data and Its Implications

Before proceeding to the various models, we first review the available data. Two groups, one from the Harvard-Smithsonian Astrophysical Observatory and the other from Imperial College have independently observed SOS in various band passes using the technique of speckle interferometry. We summarize the data from both groups[1,2,3] in Table I. Note that, using SN 1987a for comparison, SOS may have maintained roughly constant luminosity for at least $\approx 2^{1}/_{2}$ months and is very red. As seen in figure 1,the reported data on the angular separation of SOS during the period 3/25-4/14 is consistent with a proper motion in the range 0-0.6 arcsec/yr.

Based upon the data presented in Table I, we can infer certain general characteristics which any explanation of SOS must embody. Assuming a distance of 55 kpc[4] to SN 1987a, an angular separation of 60 mas corresponds to a spatial separation of $5.0 \times 10^{16} \text{ cm}$. The observed initial data on the SOS position between March 25 and April 14 suggests an early tangential propagation velocity of 1.4×10^{10}

cm/sec ($\beta\approx0.48$), assuming the SOS originated at the supernova site and was moving at constant velocity. If we use the proper motion just during this 3 week period we obtain a best fit tangential velocity of 7.5×10^9 cm/sec (($\beta\approx0.25$), although the data is still consistent with zero velocity. If we include the July possible sighting [3] then the best fit velocity is still close to this value. During the initial observation period, SOS was observed near 656 nm to be roughly 3 magnitudes dimmer than SN 1987a which translates to a luminosity of 2×10^{40} ergs/sec. If SOS maintained a constant luminosity for ≈2 months then this would imply a total energy output of at least $\approx10^{47}$ ergs. The angular resolution of these measurements is reported to be ≈7 mas, which corresponds to a scale of 6×10^{15} cm, above which an object at the distance of SN 1987a would be clearly discernable as extended. The fact that SOS is not seen to be extended then sets an upper limit on its size of a few times the resolution scale or $\approx10^{16}$ cm.

If in fact the SOS has a large tangential velocity then the emission from SOS can be strongly Doppler shifted. Thus in this case the observed frequency probably does not correspond to the frequency emitted. If the SOS motion were transverse, so that its velocity is just the maximum inferred tangential velocity, then all wavelengths are redshifted by a factor of~1.06. It is unlikely that motion of SOS along the line of sight would lead to an exact cancellation of the transverse Doppler shift.

We assume here that the velocity of SOS is less than c/2. This is done for 2 reasons: (1) taking a 1-sigma range about the transverse direction implies that the velocity vector is within 60° of the transverse plane and thus v<2v_{trans}, and (2) we find it hard to imagine a mechanism leading to sustained velocities of greater than c/2. In this case, the Doppler shift of the SOS spectrum is in the range 0.65 - 1.65. If we assume blackbody emission from SOS, the fact that SOS is apparently redder than 656 nm implies a surface temperature in the range (0.39 - 0.99)TO. The observed luminosity of SOS goes as the Doppler shift squared, and so a lower bound on the size of SOS, if it radiates as a blackbody, is $(4 - 10) \times 10^{14}$ cm.

If instead the SOS is optically thin, the required size may increase (assuming emission is not coherent) since non black-body emission is less efficient. While line emission at a higher temperature may counteract this if the emission is non-thermal, this will tend to carry only a small portion of the total black body emissivity, thus

reducing its possible impact. (Note also that the cooling time for an optically thin object is much shorter than a few months and thus this object requires an independent energy source. --see the discussion which follows). Thus, independent of the opacity of the emitting material, we estimate that 10^{14} cm $\leq R_{SOS} \leq 10^{16}$ cm.

We consider below various classes of models for SOS: a simple reflection nebula, a compact projectile emitted from SN 1987a, and a jet of matter incident on a dense gas. If the SOS is static, with constant luminosity, or moving with decreasing luminosity and/or increasing tangential size, a relativistic jet seems plausible. If the luminosity falls drastically and the SOS is static, then a reflection nebula or light echo becomes marginally possible. We suggest however that none of these models can easily accomodate the data if SOS is moving at speeds of c/4 or greater with constant luminosity. In this case the data suggest that SOS must be optically thin with an external energy source. In order to more easily explain this latter possibility, we propose that a model of an electromagnetic jet, postulated as an explanation of SS433 for similar reasons[5], can be applied to the environment of SN 1987a, yielding results which are compatible with large constant luminosity and high velocity.

II. Models

The simplest suggestion is that SOS is just a reflection nebula, or light echo, [15] which have been known to be associated with some supernova explosions. Even if SOS were stationary, such a model has obvious problems with the conservation of energy [6,7], unless there is some down-conversion of higher energy radiation. The total energy available in the optical from SN 1987a is $\approx 10^{41} \text{ergs/sec}$. At a distance of 5×10^{16} cm, in order to subtend a large enough solid angle to reflect 10% of the optical radiation SOS would have to have a size of $\approx 2\times 10^{16}$ cm., which would be detectible with the speckle interferometers used for these studies. There are also strong limits on the UV emission some time after the supernova, which preclude down-converted UV radiation as a sustained source for the SOS.[8] In addition, standard models for UV emission suggest luminosities less than 1% of that required in this case.[2] However the possibility of an unusually energetic early UV burst cannot be ruled out, as UV measurements did not begin until 14 hours after optical

discovery.[9] In this case, the luminosity of the SOS would have to fall dramatically after the initial observation. Any proper motion of this object would also argue strongly against the reflection nebula scenario since it is difficult to imagine such a nebula moving at $\approx c/4$ and expanding in such a way as to keep constant luminosity.

Next we consider whether SN 1987a might spit out a dense fragment of material with velocity ≥c/4 in the final stages of core collapse.[16] Energetically, this is not impossible as the kinetic energy of a .1 Mo fragment moving at c/4 is 6×10^{51} ergs, roughly 2% of the total binding energy available in the collapse. The first difficulty with this model is devising a method by which the ejected material would maintain a such high velocity. While the rotation or orbital velocity at the surface of a neutron star could easily be c/4, one must ensure that the ejected material is not decelerated as it escapes from the neutron star. Presumably, a 3-body interaction is needed. The next difficulty is the method in which the projectile generates the observed luminosity. If the object originates in the environment of the neutron star, it must be compact in order to avoid being ripped apart by tidal forces. On the other hand the projectile must grow significantly if it is to produce the observed luminosity. A object the size of a neutron star at such a luminosity would be very hot (≈108 K) and thus a strong UV source rather than very red. One then faces the problem of allowing a dense fragment to grow to red giant size in such a way as to allow it to remain intact near the neutron star in spite of tidal effects. The object will be composed of neutron-rich material and thus, as it expands, should become a strong gamma-ray source due to the formation of heavy elements. For example, 0.07 M_O of ⁵⁶Ni is consistent with the light curve from SN 1987a and would produce detectable gamma-rays at 847 keV even after waiting 600 days for the supernova to become optically thin[10]. Presumably the projectile has no atmosphere to shield these gamma rays and thus would be a bright gamma-ray. source as well. UV observations of SN 1987a already indicate a possible exponential luminosity tail which one would expect from down conversion of energy released in radioactive decays [8,9] Presumably these observations should also be sensitive to SOS. If the mechanism by which a self-contained projectile maintains such a high luminosity for an extended period of time is via radioactive decays this constraint is even more difficult to satisfy. The fine-tuning of the projectile model seems to

make it at best unlikely.

One might expect that a more plausible model involves the possibility that the SOS is associated with jet phenomena. In this case, large amounts of energy could be transferred from SN 1987a to regions progressively farther away [7] A luminosity in excess of $\approx 10^{40}$ ergs/sec could be carried by a jet of relativistic particles tapping the spinning neutron star's energy. Nevertheless, while fine tuning the background matter density may make a jet scenarios possible, one has problems in general dissipating a significant fraction of the jet energy realistically in optical emission, and still maintaining what may be a roughly constant velocity, a consistently high luminosity, and a size consistent with the limits given earlier. Either (1) the luminosity results from the deposition of all the jet energy, or (2) it results from the deposition of only part of its energy. In case (1), the medium will be optically thick and the luminous region will not move at high velocity. To stop a 30 MeV/particle jet of protons requires a column density of ≈10 gm/cm², about 10 times that necessary to absorb optical radiation, making the medium optically thick. If the region is optically thick, then from our previous analysis, it must be very large and it is difficult to imagine a mechanism for moving it at c/4, since the diffusion velocity of the light itself is smaller than this. Thus, even if the material which stops the jet gets blown away in the process, one would expect either smaller velocities. and/or an observably increasing luminosity and size for the deposition region. In case (2), the jet must be energetic enough so that even a fraction of its energy will result in the observed SOS luminosity. In this case, one might envisage a mechanism which allows the region of energy deposition to move outwards--for example if the luminosity were generated as momentum is equilibrated between the head of the jet and the background medium. [7] Besides the fact that the jet must then be extremely energetic, and the fine tuning of the background density which is required for this scenario to be consistent, one has the problem of converting the kinetic energy deposited in the medium into optical luminosity (in the red) with high efficiency. The rather extreme requirements on the background medium density and the jet luminosity which result if the observed luminosity comes from a region at the jet's head are reduced if one supposes the jet to lose energy throughout its length. However, in this case, one must ensure that the medium into which energy is being

deposited is being removed as the jet passes through it. Otherwise, the size of the emitting region will grow and its luminosity will increase monotonically with time. One must then arrange that the medium is dissipated as the jet passes through. Finally, one has the standard problem of envisaging a "nozzle-like" emission mechanism which ensures that angular spreading of the jet is small. None of these objections alone is fatal, so that a relativistic jet scenario remains plausible. Nevertheless, as discussed above, the requirements are rather stringent indeed. If the velocity and luminosity of SOS remain constant then strict requirements on the background density result, as well as the creation of a mechanism to convert the energy efficiently into optical radiation.

In an effort to alleviate to some degree these stringent requirements we have postulated and alternative type of jet, similar to that proposed[5] to explain the energetic source SS433. In this case, again in order to explain high velocity and high luminosity in several bands, it was proposed that the power source derive its energy from magnetic field energy, built up as field lines connected to an outer rotating accretion disk wrap up around an inner black hole. We suggest that this model may be adapted to the environment of a Type II supernova[11] and perhaps explain the SOS phenomena. In this case, magnetic field energy will be built up as field lines trapped in the non-rotating outer shock envelope wrap up around the rapidly rotating neutron star formed in SN1987a. Eventually a narrow "magnetic jet" could break through the shock envelope and travel out with velocity a substantial fraction of the speed of light. As it propagates out, its field energy is dissipated through resistive instabilities into current lines with a very high clumping factor, and this could result in the observed parameters of SOS. In what follows, we demonstrate that rough estimates for this mechanism yield values which are consistent with the present data, and allow the possibility of constant luminosity and velocity for an extended period.

If there exists a weak dipole magnetic field of ≈ 1 gauss at the presupernova surface (radius of $\approx 10^{12}$ cm) and this flux threads the $\approx 10^6$ cm. 1.4 M_O core after collapse, then the canonical B-field of $\approx 10^{12}$ gauss will result at the neutron star surface. The field lines, roughly radial as the initial shock separates from the collapsing core, remain contiguous if the medium between the two surfaces is highly

conducting [12]. This will be the case if an electron current density of at least $J_e = n_e$ $v_{drift} \approx B/R$ is present. This implies that $n_e \ge 4 \times 10^{14}$ cm⁻³ at the neutron star surface. Since B \approx 1/R², then at larger radii we have the constraint $n_e \ge 4 \times 10^{14}$ R₆-3 cm-3 (where R_6 is in units of 10^6 cm, the neutron star radius). This requirement is by no means prohibitive. If the electron density is sufficiently high, then as the inner neutron star spins up to a frequency ω , the field lines, anchored in the much more slowly rotating ejected envelope, will begin to twist at nearly ω. The high conductivity assures that the field lines will not cross, i.e. they all make the same number of turns, so that the pitch $\mu=B_{\theta}/rB_z$ is conserved (where z is the rotation axis, θ is the rotation angle, and r the cylindrical radius). Assuming for simplicity that the magnetic dipole axis and the rotation axis are collinear and that the dipole flux through one pole of the neutron star maps roughly uniformly through the expanding shell at radius R_S such that the radial flux is conserved, then $B_z(R_s) \approx 10^{12}$ $(R_{ns}/R_s)^2$ gauss. This field is essentially force-free (i.e. $J \times B = 0$) since any locally confined pressure can escape along the field lines to either surface and the field configuration tends to efficiently exclude matter. This can result in a high hydromagnetic speed at density ρ , of $v_H=(B^2/8\pi\rho)^{1/2}$. For $v_H \ge c/4$, the density must be less than $\approx 400 \text{ B}_{12} \text{ g cm}^{-3}$ (where B_{12} is the magnetic field strength in units of 10¹² gauss), a condition easily met exterior to the neutron star.

The twisting of field lines will take place inside the light cylinder. The stresses due to the presence of matter will result in the actual velocities normal to the field lines being less than c. Thus the field lines will helically wrap around the rotation axis inside the light cylinder as the neutron star rotates, resulting in two helical flux tubes extending out from the neutron star to the light cylinder, with toroidal flux being added at a rate $\phi = (\omega/2\pi) \varphi_z \approx (\omega/2\pi) R_{ns}^2 B_z(R_{ns})$. This toroidal flux will spread out to fill the the region inside the expanding shock. If we make the rough assumptions of an r^{-1} dependence for B_θ , and that the shock envelope can be treated as a cylinder of scale R_s , then the total toroidal flux will have a value of

$$\phi_{\theta} \approx R_s^2 B_{\theta}(R_s) \ln (R_s/R_{ns}), \tag{2.1}$$

where $B_{\theta}(R_s)$ is the value of the toroidal field at the shock surface. Since this total

toroidal flux is supplied by the twisting of the field lines, we also know that $\phi_0 \approx (\omega/2\pi)\phi_z t$. Therefore,

$$B_{\theta}(R_s) \approx (\omega/2\pi)\phi_z t/[R_s^2 \ln (R_s/R_{ns})], \qquad (2.2)$$

where t is the time elapsed since formation of the shock.

The total toroidal field energy inside the shock is given by (using the same approximations which lead to (2.1))

$$W_{\theta} = \int (B_{\theta}^{2}/8\pi) dV \approx \frac{1}{4} (B_{\theta}(R_{s}))^{2} R_{s}^{3} \ln (R_{s}/R_{ns}). \tag{2.3}$$

Combining (2.2) and (2.3), and letting $R_s=v_st$, we get,

$$W_{\theta} \approx 10^{41} B_{12}^2 \omega_{100}^2 R_6^4 t_{\text{sec}} / [v_8 \ln (R_s / R_{\text{ns}})] \text{ ergs}$$
 (2.4)

where ω_{100} is the neutron star rotation rate in units of 100 rad/sec, v_8 is the expansion velocity of the shock in units of 10^8 cm/sec and t is measured in seconds.

Eventually, when its energy density per unit solid angle in the flux tube is equal to that of the shock, we expect that the flux tube will break out of the region inside the shock. If $B_{\theta} \approx 1/r$, we expect that most of the field energy will be concentrated in an azimuthal angle around the rotation axis of $\approx [\ln(R_s/R_{ns})]^{-1}$, which translates into a solid angle $\approx 10^{-4}$. The energy carried by the ejecta is $\approx 10^{51}$ ergs and therefore the breakout time is $\approx 10^6$ sec. In this time the flux tube has obtained a total toroidal field energy of $> 10^{47}$ ergs, and the radius of breakout is $\approx 10^{14}$ cm (assuming unit values for the parameters in (2.4)).

After breakout, the flux tube, with a radial size of order of the light cylinder moves away from SN 1987a at roughly its hydromagnetic velocity, which can easily approach c, while still being fed toroidal flux, and thus energy, at the same rate of $\approx [B_z(R_{ns})]^2 \omega R_{ns}^3/8\pi \approx 10^{43}$ ergs/sec. This can continue until it detaches from the neutron star, when the electron current density inside the light cylinder is no-longer sufficient to keep the field lines from crossing (i.e it becomes "current carrier starved") The detachment time is very model dependent and cannot be simply

estimated.

The dissipation of this helical field energy is determined by the growth of resistive instabilities similar to those seen in laboratory plasmas[13]. The growth time scale associated with such instabilities is $\tau_{inst} \approx \tau_H^{2/5} \tau_H^{3/5}$, where τ_H is the hydromagnetic time scale, r/v_H, and τ_n is the resistive time $4\pi r^2/\eta$, expressed in terms of the resistivity η . Note that as we stated earlier, when the flux tube is inside the shock, the instability time is long compared to the flux generation rate ω . Once outside the shock, the instability decay time for a flux tube on the scale of the light cylinder, ($r \approx 10^9$ cm), is about 10^7 seconds. Over this time the field is expected to pinch into many filaments, observed in laboratory pinches [14]. The clumping factor can by large, exceeding 107. In this way the magnetic energy is radiated from a hot. locally compressed plasma, resulting in the observed optical luminosity. The emission from thin filaments allows the macroscopic emitting region to be optically thin, with emission from the locally optically thick filament surfaces. It is quite possible that a speed of c/4 could be accommodated by an eventual steady-steady situation, with flux continuing to be transferred to the expanding helical flux tube. In this case we eventually expect to see the SOS as a strong radio source.

The magnetic flux tube scenario described above has the ability to accomodate large velocities and maintain optical luminosities in excess of 10⁴⁰ erg/sec for at least 10⁷ seconds. At first glance one might expect two lobes of emission on either side of SN 1987a.. However, it is not difficult to imagine anisotropies which allow preferential breakthough on one side of the shock, or a luminosity for the second flux tube which is below detectable levels. We present this model to demonstrate its potential consistency both with the present data and with a possible continued large velocity and luminosity for the SOS. It should not be taken as a claim that such a scenario actually provides a good description of the early evolution of the supernova. More detailed calculations will be required before this assertion can be made. In the meantime, the difficulties of each of the earlier scenarios for explaining the SOS, especially if it maintains constant luminosity and high velocity, suggest that this possibility be further explored.

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Figure Captions:

Figure 1: The reported angular separations of the companion to SN 1987a are plotted as a function of the time in days following the observation of the supernova explosion. The last point, starred and placed in a box, is based on tentative reports based on noisy data which is not fully analyzed. The curves labelled i, ii, iii, represent best fits to the proper motion of the SOS under the following conditions; i. Only the first three data points are chosen, and it is assumed that the SOS originates at the supernova site.; ii. Only the first three data points are chosen, and the latter assumption in (i) is not used; iii. All four data points are chosen, and the latter assumption in (i) is not used. Also shown is the line representing zero proper motion, which falls between the error bars in all three initial sightings.

Table 1: Data on the Companion to SN 1987a

Group	Date	$\Delta \Theta$	Δm_{656}^{a}	Δm_{588}	Δm_{533}	Δm_{450}	1987a m_{vis}
CFA ^b	March 25 April 2	59 ±8	2.7 ±0.2	n.o. ^c	not seen 3.0 ±0.5	not seen 3.5-4.0	4.0 3.8
IC^d	April 14	74 ±8	≈3	detected	n.o.	n.o.	3.5
CFA	June 3		not seen	n.o.	nots seen	not seen	3.1
CFA	July 2	*≈128 ^b	*≈3.1		*3.5		4.3

a. Magnitude relative to SN1987a. Measurements within 10 nm of indicated value. b. Harvard-Smithsonian Center for Astrophysics group

c. not observed

d. Imperial College group.* unconfirmed --data analysis inconclusive [3].

